`

TOPOLOGICAL STRUCTURES ON LA Γ-SEMI GROUPS

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Abstract: In this paper, the Γ-ideal theory of LA Γ-semi groups are introduced in the topological spaces. We also studied some of the properties related to Γ- ideals of intra-regular and anti-rectangular LA Γ-semi groups. Also we have proved that the set $C(M_B)$ of all bi-ideals of an LA-Γ semi group M with left identity form an LA-Γ semi group and every bi-ideal of M is prime if and only if it is idempotent and the set of bi-ideals of M is totally ordered under inclusion. For any two bi-ideals B_1 and B_2 of an intra-regular LA Γ-semi group M with left identity, we have proved that $B_1 \cup B_2$ is a bi-ideal of M. Also a left ideal P of M is quasi-prime if and only if $(M\Gamma a)\Gamma b \subseteq P \Rightarrow a \in P \ or \ b \in P$.

Keywords: LA Γ-semi group, Intra-Regular LA Γ-semi group, Anti-rectangular LA Γ-semi group, Bi Γ - ideals and prime, semi prime bi Γ - ideals.

1. INTRODUCTION AND PRELIMINARIES

As a generalization of a semigroup, Sen [1], introduced the notion of Γ-semigroup in 1981 and developed some theory on Γ-semigroups [2], [3].Chinram R and Jirojkul, Chinram [9], extended many classical notions of semi groups to Γ-semi groups. Qaiser Mushtaq,Madad Khan And Kar Ping Shum[5] discussed the concept of bi-ideals and left ideals of an LA semigroups.A left almost Γ-semi group, abbreviated as an LA Γ-semigroup, is an algebraic structure midway between a groupoid and a commutative semigroup. An LA Γ-Semigroup is an algebraic structure satisfying left invertive law $(a\alpha b)\beta c = (c\alpha b)\beta a$ for all $a, b, c \in M, \alpha, \beta \in \Gamma$. Every LA Γ-Semigroup satisfies the medial law $(a\alpha b)\beta(c\gamma d) = (a\alpha c)\beta(b\gamma d)$ for all $a, b, c, d \in M, \alpha, \beta, \gamma \in \Gamma$.

Definition 1.1: An LA Γ-Sub semigroup B of M is said to be a bi-ideal of M if $(B\Gamma M)\Gamma B \subseteq B$.

Example 1.2: Let $M = Z$, the set of integers and $\Gamma = \mathrm{N}$,the set of natural numbers, clearly $(M, \Gamma, \cdot)$ is a LA Γ-Semigroup under usual multiplication. The set $B = 2Z$, the set of even integers is a bi-ideal of M as $(2Z\Gamma Z)\Gamma 2Z \subseteq 2Z$. In general $B = nZ$ where n is an integer are bi-ideals of $M = Z$.

Example 1.3: Let $M = Z$, the set of integers and $\Gamma = \mathrm{N}$,the set of natural numbers, then the set $B = N$,is not a bi-ideal of M as $(N\Gamma Z)\Gamma N \nsubseteq N$.

Definition 1.4: A bi-ideal B of an LA Γ-semi group M is called a prime bi-ideal if $B_1 \Gamma B_2 \subseteq B \Rightarrow B_1 \subseteq B \; or \; B_2 \subseteq B$ for any bi-ideals B_1 and B_2 of M.

Definition 1.5: A bi-ideal B of an LA Γ-semi group M is called a semi-prime bi-ideal if $B_1 \Gamma B_1 \subseteq B \; or \; {B_1}^2 \subseteq B \Rightarrow B_1 \subseteq B$ for any bi-ideal B_1 of M

Definition 1.6: A semi lattice is a structure $M = < M, \cdot >$ where $' \cdot '$ is an infix binary operation called semi lattice operation such that (i) $\cdot$ is Associative. (ii) $\cdot$ is commutative. (iii) $\cdot$ is idempotent.

Definition 1.7: A bi-ideal B of an LA Γ-semi group M is called prime bi-ideal if $B_1 \Gamma B_2 \subseteq B$ implies either $B_1 \subseteq B \; or \; B_2 \subseteq B$ for every bi-ideals $B_1 \; and \; B_2$ of M.The set of bi-ideals of M is totally ordered under inclusion if for all bi-ideals I, J of M either$I \subseteq J \; or \; J \subseteq I$.

Definition 1.8: An element $'a'$ of an LA Γ-semi group M is called intra-regular if there exists elements $x, y \in M \; and \; \alpha, \beta \in \Gamma$ such that $a = (x\alpha(a\beta a))\gamma y$.If every element of M is intra-regular then M is called intra-regular LA Γ-semi group.

Definition 1.9: Let P be a left ideal of an LA Γ-semi group M, Then P is called quasi-prime if $A\Gamma B \subseteq P \Rightarrow A \subseteq P \; or \; B \subseteq P$ for any two left ideals A and B of M.

Definition 1.10: An LA Γ-semi group M is said to be anti-rectangular if $a = (b\alpha a)\beta b$, for all $a, b \in M \; and \; \alpha, \beta \in \Gamma$.It is straight forward to see that $M = M\Gamma M$.

2. RESULTS AND DISCUSSIONS

Theorem 2.1: If M is an LA Γ-Semigroup with left identity $'e'$, Then the laws hold for any $a, b, c, d \in M \; and \; \alpha, \beta, \gamma \in \Gamma$ $i). (a\alpha b)\beta(c\gamma d) = (d\alpha b)\beta(c\gamma a)$ and $ii). a\alpha(b\beta c) = b\alpha(a\beta c)$.

Theorem 2.2: If I is a left ideal of an LA Γ-Semigroup $'M'$ with left identity, Then $I\Gamma I = I^2$ becomes an ideal of M.

Theorem 2.3: Let M be an LA Γ-Semigroup. Then each right ideal of M is a bi-ideal of M.

Theorem 2.4: If M is an LA Γ-Semigroup with left identity $'e'$ and if B is a bi-ideal of M then B^2 is also a bi-ideal of M.

Proof: Let M be an LA Γ-Semigroup with left identity $'e'$ and B is a bi-ideal of M.

Consider $\big((b_1\alpha b_2)\beta m\big)\gamma(b_3\delta b_4) = ((b_1\alpha b_2)\beta b_3)\gamma(m\delta b_4) = ((b_1\alpha b_2)\beta b_3)\gamma((b_4\delta m)\eta e) =$

$((b_1\alpha b_2)\beta(b_4\delta m))\gamma(b_3\eta e) = \left(\left((b_4\delta m)\alpha b_2\right)\beta b_1\right)\gamma(b_3\eta e) = \left((b_3\eta e)\beta b_1\right)\gamma\left((b_4\delta m)\alpha b_2\right) \in B\Gamma B = B^2 \Rightarrow ((B\Gamma B)\Gamma M)\Gamma(B\Gamma B) \subseteq B\Gamma B \Rightarrow B\Gamma B = B^2$ is a bi-ideal of M. Where $b_1, b_2, b_3, b_4 \in B, m \in M\ and\ \alpha, \beta, \gamma, \delta, \eta \in \Gamma$.

Also consider $x \in B^2 = B\Gamma B \Rightarrow a\alpha b \Rightarrow x = e\beta(a\alpha b) \in M\Gamma(B\Gamma B) \Rightarrow x \in M\Gamma(B\Gamma B) \Rightarrow B\Gamma B \subseteq M\Gamma(B\Gamma B)$. Again $m\alpha(b_1\beta b_2) = (e\gamma m)\alpha((b_2\delta b_1)\beta e) = (e\alpha(b_2\delta b_1)\beta)(e\gamma m) = (b_2\delta b_1)\beta m \in (B\Gamma B)\Gamma M \Rightarrow M\Gamma(B\Gamma B) \subseteq (B\Gamma B)\Gamma M \Rightarrow M\Gamma B^2 \subseteq B^2\Gamma M$ And $(b_1\alpha b_2)\beta m = (m\alpha b_2)\beta b_1 = m\alpha(b_2\beta b_1) \in M\Gamma(B\Gamma B) \Rightarrow B^2\Gamma M \subseteq M\Gamma B^2 \Rightarrow B^2\Gamma M = M\Gamma B^2$.

Theorem 2.5: The intersection of any two bi-ideals of a LA-Γ semi group M is either empty or a bi-ideal of M

Theorem 2.6: Every prime bi-ideal is a semi prime bi-ideal.

Theorem 2.7: Intersection of prime bi-ideal of an LA-Γ semi group M is also a prime bi-ideal.

Theorem 2.8: Intersection of prime bi-ideals of LA-Γ semi group M is a semi prime bi-ideal.

Theorem 2.9: If T is a left ideal of an LA-Γ semi group M with left identity $'e'$ then, $T^2\Gamma M = M\Gamma T^2 = T^2$.

Proof: If T is a left ideal of a LA-Γ semi group M, then we have proved that $T\Gamma T$ is an ideal. $\therefore M\Gamma(T\Gamma T) \subseteq T\Gamma T\ and\ (T\Gamma T)\Gamma M \subseteq T\Gamma T$. Now let $x \in T\Gamma T \Rightarrow x = a\alpha b\ , a, b \in T, \alpha \in \Gamma$. $\Rightarrow x = e\beta(a\alpha b) \in M\Gamma(T\Gamma T)\ \ where\ \beta \in \Gamma. x \in M\Gamma(T\Gamma T) \Rightarrow T\Gamma T \subseteq M\Gamma(T\Gamma T)$.

$\therefore M\Gamma(T\Gamma T) = T\Gamma T\ \ or\ \ M\Gamma T^2 = T^2$. Also let $x \in T\Gamma T \Rightarrow x = a\alpha b\ \ where\ a, b \in T, \alpha \in \Gamma$. $\Rightarrow x = e\beta(a\alpha b) = (e\beta a)\alpha b = (b\beta a)\alpha e \in (T\Gamma T)\Gamma M. \therefore x \in (T\Gamma T)\Gamma M \Rightarrow T\Gamma T \subseteq (T\Gamma T)\Gamma M. \therefore T\Gamma T = (T\Gamma T)\Gamma M\ \ or\ \ T^2 = T^2\Gamma M$.

Theorem 2.10: If T is left ideal of an LA-Γ semi group M then $T^2\Gamma B = (T\Gamma T)\Gamma B$ is a bi-ideal.

Proof: Consider $\left(\left((T\Gamma T)\Gamma B\right)\Gamma M\right)\Gamma\left((T\Gamma T)\Gamma B\right) = ((T\Gamma T)\Gamma(B\Gamma M))\Gamma((T\Gamma T)\Gamma B) = ((T\Gamma T)\Gamma(T\Gamma T))\Gamma((B\Gamma M)\Gamma B) \subseteq (T\Gamma T)\Gamma B. \therefore \left(\left((T\Gamma T)\Gamma B\right)\Gamma M\right)\Gamma\left((T\Gamma T)\Gamma B\right) \subseteq (T\Gamma T)\Gamma B$.

Since T is left ideal $\Rightarrow T\Gamma T$ is also a left ideal. Also every left ideal is a sub Γ-semi group. $\therefore T\Gamma T$ is a sub Γ-semigroup $\Rightarrow (T\Gamma T)\Gamma(T\Gamma T) \subseteq T\Gamma B$ and B is a bi-ideal $\Rightarrow (B\Gamma M)\Gamma B \subseteq B$ and $\left((T\Gamma T)\Gamma B\right)\Gamma\left((T\Gamma T)\Gamma B\right) = ((T\Gamma T)\Gamma(T\Gamma T))\Gamma(B\Gamma B) \subseteq (T\Gamma T)\Gamma B. \therefore (T\Gamma T)\Gamma B$ is a sub Γ-semi group $\Rightarrow (T\Gamma T)\Gamma B$ is a bi-ideal of M.

Theorem 2.11: The product of two bi-ideals of an LA-Γ semi group M with left identity is also a bi-ideal of M.

Proof: Let B_1 and B_2 be any two bi-ideals of an LA-Γ semi group M with left identity. Consider,$\big((B_1\Gamma B_2)\Gamma M\big)\Gamma(B_1\Gamma B_2) = ((B_1\Gamma B_2)\Gamma(M\Gamma M))\Gamma(B_1\Gamma B_2)$
$= ((B_1\Gamma M)\Gamma B_1)\Gamma((B_2\Gamma M)\Gamma B_2) \subseteq B_1\Gamma B_2$. Also $(B_1\Gamma B_2)\Gamma(B_1\Gamma B_2) = (B_1\Gamma B_1)\Gamma(B_2\Gamma B_2) \subseteq B_1\Gamma B_2$. ∴ The product $B_1\Gamma B_2$ is a bi-ideal of M. This theorem leads to an easy generalizations. i.e., if $B_1, B_2, \dots\dots\dots\dots\dots . B_n$ are bi-ideals of a LA-Γ semi group M with left identity, then $(\dots\dots\dots(B_1\Gamma B_2)\Gamma B_3 \dots\dots\dots)\Gamma B_n$ and $(\dots\dots\dots\big({B_1}^2\Gamma {B_2}^2\big)\Gamma {B_3}^2 \dots\dots\dots)\Gamma {B_n}^2$ are bi-ideals of M.

Theorem 2.12: The set $C(M_B)$ of all bi-ideals of an LA-Γ semi group M with left identity form an LA-Γ semi group.

Proof: Let $C(M_B) = \{B/\ B\ \ is\ \ a\ \ bi-ideal\ of\ M\ \}$. Clearly $C(M_B) \neq \emptyset$ as M itself is a bi-ideal.

Let $B_1, B_2\ and\ B_3 \in C(M_B)$, Then $B_1\Gamma B_2 \in C(M_B)$. Let $x \in (B_1\Gamma B_2)\Gamma B_3 \Rightarrow x = a\alpha(b\beta c)$ where$a \in B_1, b \in B_2, c \in B_3\ \ and\ \alpha,\beta \in \Gamma$. ∴ $x \in B_1\Gamma(B_2\Gamma B_3) \Rightarrow (B_1\Gamma B_2)\Gamma B_3 \subseteq B_1\Gamma(B_2\Gamma B_3)$. Similarly we shall show that $B_1\Gamma(B_2\Gamma B_3) \subseteq (B_1\Gamma B_2)\Gamma B_3$ ∴ $(B_1\Gamma B_2)\Gamma B_3 = B_1\Gamma(B_2\Gamma B_3)$. To prove LA Property $(B_1\Gamma B_2)\Gamma B_3 = (B_3\Gamma B_2)\Gamma B_1$. Let $x \in (B_1\Gamma B_2)\Gamma B_3 \Rightarrow x = (a\alpha b)\beta c$ where $a \in B_1, b \in B_2, c \in B_3\ \ and\ \alpha,\beta \in \Gamma$. ∴ $x = (c\alpha b)\beta a \Rightarrow x \in (B_3\Gamma B_2)\Gamma B_1 \Rightarrow (B_1\Gamma B_2)\Gamma B_3 \subseteq (B_3\Gamma B_2)\Gamma B_1$. Similarly $(B_3\Gamma B_2)\Gamma B_1 \subseteq (B_1\Gamma B_2)\Gamma B_3$.
∴ $(B_1\Gamma B_2)\Gamma B_3 = (B_3\Gamma B_2)\Gamma B_1$.∴ $C(M_B)$ is an LA Γ-semi group.

Theorem 2.13: If A and B are bi Γ- ideals of an LA-Γ semi group M with left identity, Then the following assertions are equivalent.

i) Every bi-ideal of M is idempotent.

ii) $A \cap B = A\Gamma B$ and

iii) The ideals of M form a semi lattice $(L_M, \wedge)$ where $A\wedge B = A\Gamma B$.

Proof: i) ⇒ ii).

Let A and B are any two bi-ideals of an LA-Γ semi group M with left identity. Then from i) Every bi-ideal of M is idempotent and hence from theorem 2.11, A and B are ideals of M. ∴ $A\Gamma M \subseteq A, M\Gamma A \subseteq A\ \ and\ \ B\Gamma M \subseteq B,\ M\Gamma B \subseteq B$. To prove that $A\Gamma B \subseteq A \cap B$.Let $\in A\Gamma B \Rightarrow x = a\alpha b, where\ a \in A, b \in B, \alpha \in \Gamma \quad \Rightarrow x = a\alpha b,\ where\ a \in A, b \in M, \alpha \in \Gamma \Rightarrow x = a\alpha b \in A\Gamma M \subseteq A \Rightarrow x \in A$. Also $x = a\alpha b, where\ a \in A, b \in B, \alpha \in \Gamma \ \Rightarrow x = a\alpha b, where\ a \in M, b \in B, \alpha \in \Gamma \ \Rightarrow x = a\alpha b \in M\Gamma B \subseteq B \Rightarrow x \in B$. ∴ $x \in A\ \ and\ x \in B \Rightarrow x \in A \cap B$.∴ $x \in A\Gamma B \Rightarrow x \in A \cap B \Rightarrow A\Gamma B \subseteq A \cap B$. To prove $A \cap B \subseteq A\Gamma B$. We know that $A \cap B \subseteq A\ and\ A \cap B \subseteq$

$B \Rightarrow (A \cap B)\Gamma(A \cap B) \subseteq A\Gamma B$. Since A and B are bi-ideals and intersection of bi-ideals is either empty or a bi-ideal. $\therefore A \cap B$ is bi-ideal and it is given that every bi-ideal is idempotent $\Rightarrow A \cap B$ is idempotent. $\Rightarrow (A \cap B)\Gamma(A \cap B) = A\Gamma B \Rightarrow A \cap B \subseteq A\Gamma B. \therefore A \cap B = A\Gamma B$.

ii) $\Rightarrow$ iii). Let $A \cap B = A\Gamma B$. $L_M = \{A/A \text{ is an ideal of } M\}$.For $A, B \in L_M$ define $A \wedge B = A\Gamma B$.

Consider $A \wedge (B \wedge C) = A\Gamma(B\Gamma C) = A \cap (B \cap C) = (A \cap B) \cap C = (A\Gamma B)\Gamma C = (A \wedge B) \wedge C$.

$\therefore \wedge$ is associative. Again $A \wedge B = A\Gamma B = A \cap B = B \cap A = B\Gamma A = B \wedge A$. $\therefore \wedge$ is commutative.

Also $A \wedge A = A\Gamma A = A \cap A = A$. $\Rightarrow \wedge$ is idempotent. $\therefore (L_M, \wedge)$ form a semi lattice.

iii) $\Rightarrow$ i)

Let the ideals of M $(L_M, \wedge)$ form a semi lattice where $A \wedge B = A\Gamma B$. Let A be a bi-ideal of M.Then by theorem 2.11, A is an ideal. $\therefore$ Consider $A\Gamma A = A \wedge A = A \Rightarrow A$ is idempotent. Since A is arbitrary, every bi-ideal of M is idempotent.

Theorem 2.14: Let M be an LA Γ-semi group with left identity e. Then every bi-ideal of M is prime if and only if it is idempotent and the set of bi-ideals of M is totally ordered under inclusion.

Proof: Let M be a LA Γ-semi group with left identity e.Let us suppose that every bi-ideal of M is prime. Let B be a bi-ideal of M. Then by theorem 2.14, product of two bi-ideal is also a bi-ideal $\Rightarrow B\Gamma B$ is also a bi-ideal of M.By our assumption every bi-ideal is prime$\Rightarrow B\Gamma B$ is also prime. Clearly $B\Gamma B \subseteq B\Gamma B \Rightarrow B \subseteq B\Gamma B$.Also every bi-ideal is a sub Γ-semi group $\Rightarrow B\Gamma B \subseteq B$

$\therefore B = B\Gamma B \Rightarrow B$ is idempotent.

Let B_1 and B_2 are bi-ideals of M. Then by theorem 2.13, $B_1\Gamma B_2$ is also bi-ideal of M. $\therefore B_1\Gamma B_2$ is idempotent as every bi-ideal is idempotent. By theorem 2.13, $B_1 \cap B_2 = B_1\Gamma B_2$. $\therefore B_1 \cap B_2$ is a bi-ideal of M, and by given hypothesis $B_1 \cap B_2$ is prime. $\therefore B_1\Gamma B_2 \subseteq B_1 \cap B_2 \Rightarrow B_1 \subseteq B_1 \cap B_2 \text{ or } B_2 \subseteq B_1 \cap B_2 \Rightarrow B_1 \subseteq B_2 \text{ or } B_2 \subseteq B_1$. $\therefore$ The set of bi-ideals of M is totally ordered under inclusion.

Conversely, Let every bi-ideal of M is idempotent and the set of bi-ideals of M is totally ordered under inclusion, we shall prove that every bi-ideal of M is prime. Let B_1,B_2 and B are bi-ideals of M with $B_1\Gamma B_2 \subseteq B$. Assume that $B_1 \subseteq B_2$, Since B_1 is idempotent $B_1 = B_1\Gamma B_1 \subseteq B_1\Gamma B_2 \subseteq B \Rightarrow B_1 \subseteq B$.Similarly if $B_2 \subseteq B_1 \Rightarrow B_2 \subseteq B \Rightarrow B$ is prime.

Theorem 2.15: If B_1 and B_2 are bi-ideals of an intra-regular LA Γ-semi group M with left identity, then $B_1 \cup B_2$ is a bi-ideal of M.

Proof: Consider $\big((B_1 \cup B_2)\Gamma M\big)\Gamma(B_1 \cup B_2) = (B_1\Gamma M \cup B_2\Gamma M)\Gamma(B_1 \cup B_2)$ $= (B_1\Gamma M)\Gamma(B_1 \cup B_2) \cup (B_2\Gamma M)\Gamma(B_1 \cup B_2) = (B_1\Gamma M)\Gamma B_1 \cup (B_1\Gamma M)\Gamma B_2 \cup (B_2\Gamma M)\Gamma B_1 \cup$ $(B_2\Gamma M)\Gamma B_2 \subseteq B_1 \cup (B_1\Gamma M)\Gamma B_2 \cup (B_2\Gamma M)\Gamma B_1 \cup B_2$.

Let $(b\alpha m)\beta a \in (B_1\Gamma M)\Gamma B_2$ where $b \in B_1, m \in M, a \in B_2$ and $\alpha, \beta \in \Gamma$.Since M is intra – regular for $a \in M$ there exists $x, y \in M$ and $\eta, \gamma, \delta \in \Gamma$ such that $a = (x\eta(a\gamma a))\delta y$.

Consider $(b\alpha m)\beta a = (b\alpha m)\beta\Big(\big(x\eta(a\gamma a)\big)\delta y\Big) = \big(x\eta(a\gamma a)\big)\beta\big((b\alpha m)\delta y\big) =$ $\big(a\eta(x\gamma a)\big)\beta\big((b\alpha m)\delta y\big) = (\big((b\alpha m)\delta y\big)\eta(x\gamma a))\beta a = (((b\alpha m)\delta y)\eta(x\gamma\Big(\big(x\xi(a\varepsilon a)\big)\varphi y\Big))\beta a$

where φ, ε, ξ∈ Γ

$= (\big((b\alpha m)\delta y\big)\eta\big(x\xi(a\varepsilon a)\big)\gamma(x\varphi y)))\beta a = \Big(\Big(\big(x\xi(a\varepsilon a)\big)\eta\big((b\alpha m)\delta y\big)\Big)\gamma(x\varphi y)\Big)\beta a$

$= (\Big((x\varphi y)\eta(b\alpha m)\delta\Big(y\gamma\big(x\xi(a\varepsilon a)\big)\Big)\Big)\beta a = (((x\varphi y)\eta\Big((b\alpha m)\delta y)\big)\gamma\big(x\xi(a\varepsilon a)\big)\Big)\beta a.$

$= \Big(\Big((a\varepsilon a)\eta\big((b\alpha m)\delta y\big)\Big)\gamma\big(x\xi(x\varphi y)\big)\Big)\beta a = \Big((a\varepsilon a)\eta\big((b\alpha m)\delta y\big)\gamma\big(x\xi(x\varphi y)\big)\Big)\beta a \in$ $(B_2\Gamma S)\Gamma B_2 \subseteq B_2$.∴ $(B_1\Gamma M)\Gamma B_2 \subseteq B_2$.Similarly we can show that $(B_2\Gamma M)\Gamma B_1 \subseteq B_1$.

∴ $\big((B_1 \cup B_2)\Gamma M\big)\Gamma(B_1 \cup B_2) \subseteq B_1 \cup B_2 \Rightarrow B_1 \cup B_2$ is a bi-ideal of M.

Theorem 2.16: If M is an LA Γ-semi group with left identity e, then a left ideal P of M is quasi-prime if and only if $(M\Gamma a)\Gamma b \subseteq P \Rightarrow a \in P$ or $b \in P$.

Proof: Let P be a left ideal of an LA Γ-semi group M with left identity e. Let P be quasi-prime. Then $(M\Gamma a)\Gamma b \subseteq P \Rightarrow M\Gamma((M\Gamma a)\Gamma b) \subseteq M\Gamma P \subseteq P \Rightarrow M\Gamma\big((M\Gamma a)\Gamma b\big) \subseteq P \Rightarrow$ $(M\Gamma a)\Gamma(M\Gamma b) \subseteq P \Rightarrow M\Gamma a \subseteq P$ or $M\Gamma b \subseteq P \Rightarrow e\alpha a \in P$ or $e\beta b \in P \Rightarrow a \in$ P or $b \in P \Rightarrow (M\Gamma a)\Gamma b \subseteq P \Rightarrow a \in P$ or $b \in P$.

Conversely, Let $(M\Gamma a)\Gamma b \subseteq P \Rightarrow a \in P$ or $b \in P$,To prove that P is quasi-prime ideal of M.

Let us assume that $A\Gamma B \subseteq P$,where A and B are left ideals of M such that $A \nsubseteq P$.

$\Rightarrow x \in A$ such that $x \notin P$.Now $(M\Gamma x)\Gamma y \subseteq (M\Gamma A)\Gamma B \subseteq A\Gamma B \subseteq P$ for all $y \in B$. ∴ $(M\Gamma x)\Gamma y \subseteq P$.So by hypothesis $x \notin P \Rightarrow y \in P$.∴ $y \in B \Rightarrow y \in P \Rightarrow B \subseteq P$.

Similarly if we assume that $B \nsubseteq P$,Then we can prove $A \subseteq P$.∴ $A\Gamma B \subseteq P \Rightarrow A \subseteq P$ or $B \subseteq P$.

$\Rightarrow P$ is quasi-prime ideal of M.

Theorem 2.17: If A and B are ideals of an anti-rectangular LA Γ-semi group M,Then $A\Gamma B$ is an ideal.

Proof: Consider $(A\Gamma B)\Gamma M = (A\Gamma B)\Gamma(M\Gamma M) = (A\Gamma M)\Gamma(B\Gamma M) \subseteq A\Gamma B$. Similarly $M\Gamma(A\Gamma B) = (M\Gamma M)\Gamma(A\Gamma B) = (M\Gamma A)\Gamma(M\Gamma B) \subseteq A\Gamma B \Rightarrow A\Gamma B$ is an ideal of M. Consequently if $I_1, I_2, I_3, \dots\dots\dots\dots I_n$ are ideals of M, then $(\dots\dots\dots((I_1\Gamma I_2)\Gamma I_3 \dots\dots\dots \Gamma I_n)$ and $(\dots\dots\dots\left(\left({I_1}^2\Gamma {I_2}^2\right)\Gamma {I_3}^2\right)\dots\dots\dots\Gamma {I_n}^2)$ are ideals of M and the set S_I of ideals of M form an anti-rectangular LA Γ-semi group.

Theorem 2.18: Let M be an anti-rectangular LA Γ-semi group then $S_I = \{I/I\ is\ an\ ideal\ of\ M\}$ is an anti-rectangular LA Γ-semi group.

Proof: Let M be an anti-rectangular LA Γ-semi group. Let $S_I = \{I/I\ is\ an\ ideal\ of\ M\}$

i) To prove S_I is a Γ-semi group:

Let $I_1, I_2 \in S_I$, from theorem 2.14,we see that $I_1\Gamma I_2$ is also an ideal of M.∴ $I_1\alpha I_2 \in S_I$ for α∈Γ. To prove $(I_1\alpha I_2)\beta I_3 = I_1\alpha(I_2\beta I_3)$ for $I_1, I_2, I_3 \in S_I\ and\ \alpha, \beta \in \Gamma$, Let $x \in (I_1\alpha I_2)\beta I_3 \Rightarrow x = (i_1\alpha i_2)\beta i_3$ where $i_1 \in I_1, i_2 \in I_2, i_3 \in I_3, \alpha, \beta \in \Gamma$ $\Rightarrow x = i_1\alpha(i_2\beta i_3) \Rightarrow x \in I_1\alpha(I_2\beta I_3) \Rightarrow (I_1\alpha I_2)\beta I_3 \subseteq I_1\alpha(I_2\beta I_3)$.Similarly we can show that $I_1\alpha(I_2\beta I_3) \subseteq (I_1\alpha I_2)\beta I_3$.$\Rightarrow (I_1\alpha I_2)\beta I_3 = I_1\alpha(I_2\beta I_3)$.∴ S_I is a Γ-semi group.

ii) To prove that S_I satisfies left invertive law.

Let $x \in (I_1\alpha I_2)\beta I_3$where $I_1, I_2, I_3 \in S_I$. $x = (i_1\alpha i_2)\beta i_3$ where $i_1 \in I_1, i_2 \in I_2, i_3 \in I_3, \alpha, \beta \in \Gamma$.∴ $x = (i_3\alpha i_2)\beta i_1) \Rightarrow x \in (I_3\alpha I_2)\beta I_1 \Rightarrow (I_1\alpha I_2)\beta I_3 \subseteq (I_3\alpha I_2)\beta I_1)$.

Similarly we can show that $(I_3\alpha I_2)\beta I_1 \subseteq (I_1\alpha I_2)\beta I_3$.$\Rightarrow (I_1\alpha I_2)\beta I_3 = (I_3\alpha I_2)\beta I_1$.

∴ S_I satisfies left invertive law $\Rightarrow S_I$ is a LA Γ-semi group.

iii) To prove S_I is anti-rectangular.

i.e.,$I_1 = (I_2\alpha I_1)\beta I_2, \forall\ I_1, I_2 \in S_{I,}\alpha, \beta \in \Gamma$.Let $i_1 \in I_1 \Rightarrow i_1 \in M$, for $i_2 \in I_2, (i_2\alpha i_1)\beta i_2 \in (I_2\alpha I_1)\beta I_2 \Rightarrow i_1 \in (I_2\alpha I_1)\beta I_2 \Rightarrow I_1 \subseteq (I_2\alpha I_1)\beta I_2$.Also let $x \in (I_2\alpha I_1)\beta I_2 \Rightarrow x = (i_2\alpha i_1)\beta i_2$ for $i_1 \in I_1, i_2 \in I_2, \alpha, \beta \in \Gamma$.∴ $x = (i_2\alpha i_1)\beta i_2$ where $i_1, i_2 \in M$.$\Rightarrow x = i_1 \in I_1 \Rightarrow x \in I_1$.∴ $I_1 = (I_2\alpha I_1)\beta I_2$.$S_I$ is anti-reactangular LA Γ-semi group.

Theorem 2.19: Any subset of an anti-rectangular LA Γ-semi group M is left ideal if and if it is right ideal.

Proof: Let M be an anti-rectangular LA Γ-semi group. Let $I \subseteq M$ be a left ideal.∴ $I\Gamma M \subseteq I$.

Let $x \in M\Gamma I \Rightarrow x = m\alpha i, m \in M, \alpha \in \Gamma, i \in I$. ∴ $x = \big((a\beta m)\gamma a\big)\alpha i\ \forall a \in M, \beta, \gamma \in \Gamma$.

$x = (i\gamma a)\alpha(a\beta m) \Rightarrow x \in I\Gamma M$. ∴ $x \in I\Gamma M \subseteq I \Rightarrow x \in I$.∴ $M\Gamma I \subseteq I$.

Conversely, Let I be a right ideal of M.i.e.,$M\Gamma I \subseteq I$.Let $x \in I\Gamma M \Rightarrow x = i\alpha m, m \in M, \alpha \in \Gamma, i \in I. \therefore x = \big((a\beta i)\gamma a\big)\alpha m\ \forall a \in M, \beta, \gamma \in \Gamma. \therefore x = (m\gamma a)\alpha(a\beta i) \Rightarrow x \in M\Gamma I. \therefore x \in M\Gamma I \subseteq I \Rightarrow x \in I \therefore I\Gamma M \subseteq I. \therefore I$ is a left ideal of M.

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